Overdetermined linear equations of the form $Ax = b$, where the number of rows of matrix A exceeds the number of columns, can often be solved by optimization techniques, although solutions may not be unique.

Such a linear system consisting of a m by n matrix A where $m > n$, and a m by 1 vector b as in $Ax = b$ cannot be solved uniquely, but often solutions can be found by minimizing some L_p norm of the residuals r_i such as

$$L_p = \left(\sum_{i=1}^{m} |r_i|^p \right)^{1/p}$$

where typically p can be 1, 2, or ∞. In some cases starting estimates will be required.

From the main SIMFIT menu choose [Statistics] then [Numerical analysis] and run the three options for p using the default test files `matrix.tf2` defining the 7 by 5 matrix A and `vector.tf2` containing the 7 by 1 vector $b^T = (1, 2, 3, 4, 5, 6, 7)$ as follows.

$$A = \begin{pmatrix} 1.20 & 3.60 & 1.90 & 8.50 & 3.20 \\ 4.70 & 8.85 & 9.91 & 2.50 & 8.06 \\ 6.34 & 8.12 & 5.56 & 3.45 & 7.76 \\ 3.65 & 7.78 & 3.48 & 1.15 & 6.67 \\ 3.32 & 8.83 & 4.46 & 7.82 & 4.49 \\ 3.61 & 7.82 & 1.08 & 5.22 & 6.38 \\ 6.12 & 5.51 & 8.03 & 5.61 & 4.43 \end{pmatrix}$$

L_1 norm solution to $Ax = b$
1.9514418E+00
4.2111129E-01
-5.6336298E-01
4.3037848E-02
-6.7286341E-01
objective function = 4.9251750E+00

L_2 norm solution to $Ax = b$
1.2955430E+00
7.7602676E-01
-3.3656942E-01
8.2383926E-02
-9.8542254E-01
The rank of A (from SVD) = 5
objective function = 1.0961673E+01

L_∞ norm solution to $Ax = b$
1.0529866E+00
7.4896175E-01
-2.7683128E-01
2.6138630E-01
-9.7904715E-01
objective function = 1.5226995E+00